Exam Kwantumfysica 2

Date	20 June 2013
Room	Aletta Jacobshal 03
Time	14:00 - 17:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are allowed to use the book "Introduction to Quantum Mechanics" by Griffiths
- You are *not* allowed to use print-outs, notes or other books
- The weights of the exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

1a)	7	2a)	7	3a)	7	4)	12
1b)	7	2b)	12	3b)	12		
1c)	7	2c)	7				
1d)	12						

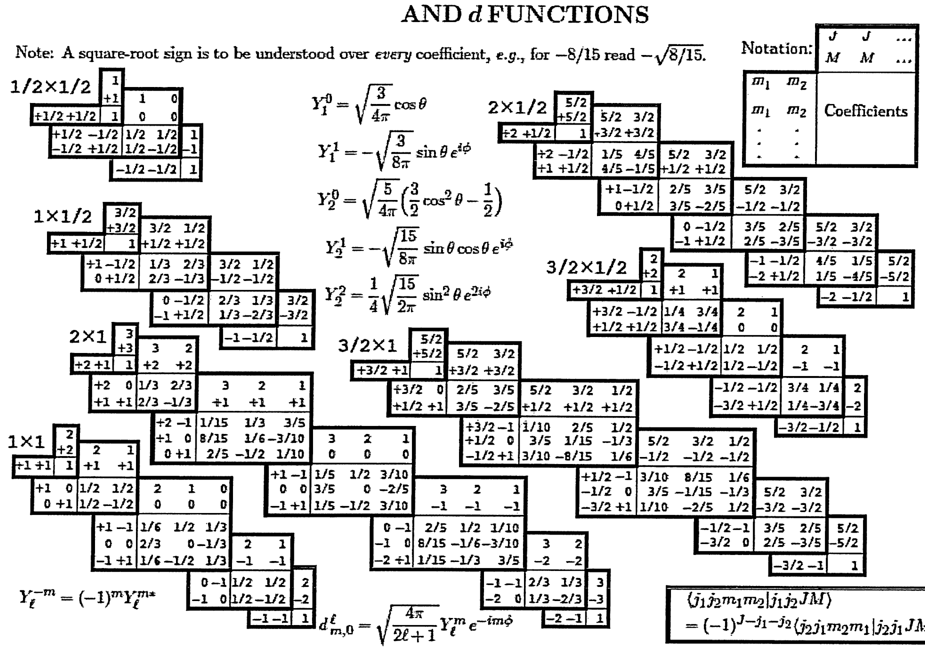
$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

Exercise 1

(a) Write down the general form of the Clebsch-Gordan decomposition of states $|j_1, j_2; j, m\rangle$ into $|j_1, j_2; m_1, m_2\rangle$ and of $|j_1, j_2; m_1, m_2\rangle$ into $|j_1, j_2; j, m\rangle$. Explain or show why the (real) Clebsch-Gordan coefficients entering in these expressions are the same. Hint: project out the coefficients.

(b) Use the table below to write down the Clebsch-Gordan decomposition of the state $|l, s; j, m\rangle = |1, \frac{1}{2}; \frac{1}{2}, -\frac{1}{2}\rangle$ and verify that acting with J_- on it gives zero.

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS



(c) Show that J_z commutes with $\vec{L}\cdot\vec{S}$ and that $\vec{J}^2$ does not commute with $L_z + 2S_z$.

(d) Consider the effect of a uniform electric field along the $\hat{z}$ direction on the four $n = 2$ levels of hydrogen. Recall that the Stark effect is described by the term $H_S = eEz$ in the Hamiltonian. Explain why for $n = 2$ the matrix elements $\langle l', m'|H_S|l, m\rangle$ take the form:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -3eEa & 0 \\ 0 & -3eEa & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

when written on the basis $e_1 = |211\rangle, e_2 = |210\rangle, e_3 = |200\rangle, e_4 = |21-1\rangle$.

Exercise 2

Consider a two-dimensional square well potential:

$$V(x,y) = \begin{cases} 0 & \text{for } 0 \le x \le a \text{ and } 0 \le y \le a \\ \infty & \text{elsewhere} \end{cases}$$

(a) The first excited state of the system is degenerate. Give its energy and the explicit expressions for the corresponding wave functions.

Next introduce the perturbation:

$$H'(x,y) = \begin{cases} kxy & \text{for } 0 \le x \le a \text{ and } 0 \le y \le a \text{ and } k > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(b) Calculate in first order perturbation theory the correction(s) to the energy level of the first excited state. You may exploit symmetry to simplify the calculation, provided you give an explanation. You may use the following integrals:

$$\int_0^a x \sin^2(n\pi x/a)dx = \frac{a^2}{4}, \quad \int_0^a x \sin(\pi x/a)\sin(2\pi x/a)dx = -\frac{8a^2}{9\pi^2},$$

where n is an integer.

(c) Explain why the degeneracy is lifted by the perturbation. Hint: consider the symmetries of V and $V + H'$.

Exercise 3

Consider the one-dimensional harmonic oscillator.

(a) Write down a trial wave function that is sure to give an upper bound to the energy of the first excited state.

(b) Find the tightest bound to the first excited energy level using this trial wave function. Explain whether the answer is as might have been expected. (Suggestion: if you do not know a proper trial wave function or if the integrals are too difficult, at least explain how you would do the calculation in principle).

Exercise 4

Consider the Hamiltonian $H = H_0 + H'(t)$, where $H'(t) = V(r)\theta(t - t_0)$ is a perturbation acting from time t_0 onwards. Consider the particular case of a two-level system consisting of states ψ_1 and ψ_2, for which $\langle\psi_i|H'(t)|\psi_j\rangle \neq 0$ only for $i \neq j$. Derive, using time-dependent perturbation theory and keeping terms at most quadratic in the perturbation, what is the probability to be in state 2 as a function of time if the system is in state 1 before t_0.